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Buckling of nanotubes under compression considering surface effects

J. E. Jam*
A. T. Samaei

Composite Materials and Technology Center, MUT, Tehran, Iran.

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ABSTRACT

In this paper, the modified Euler-Bernoulli beam model is presented to examine the influence of surface elasticity and residual surface tension on the critical force of axial buckling of nanotubes in the presence of rotary inertia. An explicit solution is derived for the buckling loads of microscaled Euler beams considering surface effects. The size-dependent buckling behavior of the nanotube due to surface effects is well elucidated in the obtained solutions. The critical forces are evaluated for axial buckling of cantilever beams. The results are compared with those corresponding to the classical beam model. The influences of the surface effects on the critical forces are discussed in detail.

Keywords: *Nanotubes; Nanowire; Surface effect; Buckling; Euler-Bernoulli beam model.*

INTRODUCTION

In the last few years it has been received increasing attentions for the importance of surface effects on the mechanical properties of micro/nanoscaled structures due to their potential applications of micro/nano-beam or tube-like structures in micro/nano-electromechanical systems (MEMS, NEMS) and atomic force microscopy (AFM) devices with high sensitivity and high frequency [1,2]. In contrast with large scales, the surface effects cannot be ignored in atomistic scales because the volume ratio of the surface region to the bulk is large [3]. For aforementioned applications and reason, understanding the exact characterization of the mechanical properties of these micro/nanoscaled structures including surface effects are of fundamental concern in design and predicting performance of the devices. In addition, some experimental techniques and atomistic simulations are developed to measure the material properties for the purpose of disclosing the phenomena of surface effects [4].

* Corresponding author:
J. E. Jam
Composite Materials and Technology Center, MUT, Tehran, Iran.
Tel +98 21 22935300
Fax +98 21 22936578
Email jejam@mail.com

The surface elasticity theory has been adopted to illustrate various size-dependent phenomena at the micro/nanoscale, and the predictions fit well with atomistic simulations and experimental measurements [5,6]. He and Lilley [6] studied the elastic behavior of static bending of nanowires considering surface effects and compared their results by experiment. Recently beam models are used to explain vibration of many micro/nanoscaled structural elements such as beams, wires, and tubes [7-9]. For example, Abbasion and *et al*. [7] studied the free vibration of Timoshenko microbeams in the presence of surface elasticity and explored the differences of results compared to the classic theory. For the applications of nanotubes as sensors, the axial buckling is of special interest. Recently, Wang and Feng [9,10] explored the surface effects on the axial buckling of nanowires using refined Euler and Timoshenko beam theory. The purpose of this paper is to investigate the coupled effects of surface elasticity, residual surface stress and rotary inertia on the axial buckling behavior of nanotubes. The Euler beam theory integrated with the surface elasticity model is applied to derive the analytical solutions of the critical buckling force of the nanotubes.

EXPERIMENTAL

Problem solution

Generally, surface effects on the mechanical behavior of nanoscaled materials and structures can be examined by considering surface energy and/or surface stresses. According to Gibbs [11] and Cammarata [12], the surface stresses tensor $\sigma^s_{\alpha\beta}$, $(\alpha,\beta = 1,2)$, is related to the surface energy density γ through the surface strain tensor $\varepsilon^s_{\alpha\beta}$ by

$$\sigma^s_{\alpha\beta} = \gamma\delta_{\alpha\beta} + \frac{\partial\gamma}{\partial\varepsilon^s_{\alpha\beta}} \qquad (1)$$

The one-dimensional and linear form of equation (1) reads

$$\sigma^s = \tau^0 + E^s\varepsilon \qquad (2)$$

Where τ^0 and E^s are the residual surface tension under unstrained condition and surface Young's modulus which can be determined by atomistic simulations or experiments [13], respectively. Axial buckling of a nanotube having length L and inner and outer radii R_i and R_o is shown in Figure 1.

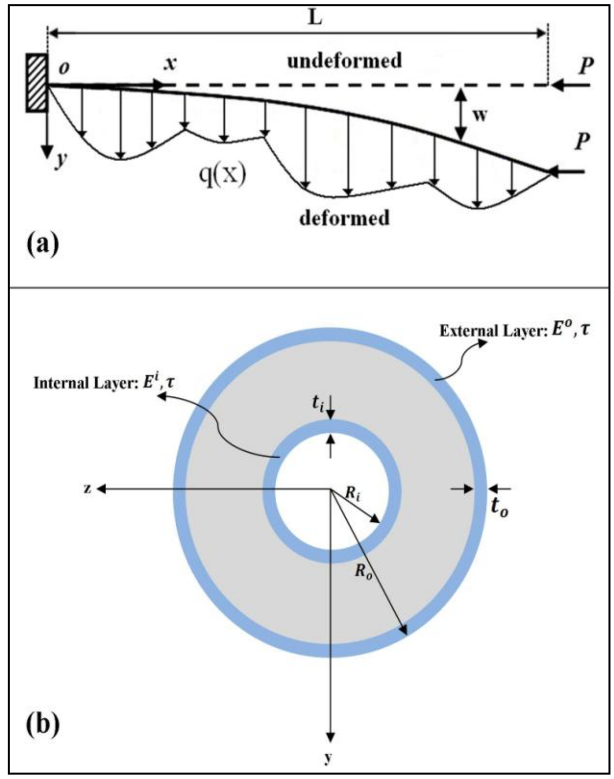

Fig. 1. (color online) (a) Buckling of a nanotube under uniaxial compression,(b) Cross-section view of the tube with two internal and external thin layers.

RESULTS AND DISCUSSION

The Young's modulus and mass density of the nanotube are denoted as E and ρ, respectively. The effect of surface elasticity, expressed by the second term in equation (1) or (2), can be modeled by two thin layers with surface elasticity modulus of E^i and E^o. Denote the thickness of inner and outer layer by t_i and t_o, respectively. In order to regenerate the idealized surface with zero thickness assumed in the surface elasticity theory, we can let t approach zero while keeping $E^o t_o$ and $E^i t_i$ as the constant of surface stiffness E^s [13]. Therefore, the effects of surface elasticity on the bending of a beam can be admitted by the following effective flexural rigidity [8]:

$$(EI)^* = \frac{\pi E(R_o^4-R_i^4)}{4} + \pi E^s(R_o^3 + R_i^3) \qquad (3)$$

When the radii are of nano orders, the effect of E^s cannot be ignored. The effect of the residual surface stress on the nanotube is determined by the Laplace-Young equation. The stress jump across each surface $\langle\sigma_{ij}^+ - \sigma_{ij}^-\rangle$ is related to the curvature tensor $(\kappa_{\alpha\beta})$ of the surface as follows:

$$\langle\sigma_{ij}^+ - \sigma_{ij}^-\rangle n_i n_j = \sigma_{\alpha\beta}^s \kappa_{\alpha\beta} \qquad (4)$$

Where σ_{ij}^+ and σ_{ij}^- are the stresses above and below the surface, respectively, n_i is the unit vector normal to the surface. Therefore, the Laplace-Young equation in equation (4) predicts the distributed transverse loading $q(x)$ on the beam [8]

$$q(x) = H\kappa \approx H\frac{\partial^2 w}{\partial x^2} \approx 4\tau(R_o + R_i)\frac{\partial^2 w}{\partial x^2} \qquad (5)$$

Where Ha constant depending on the residual is surface tension and w denotes transverse displacement along z axis. The governing equation of the nanotube concerning both surface elasticity and residual surface tension becomes

$$\left[\frac{\pi E}{4}(R_o^4 - R_i^4)\right]\frac{\partial^4 w}{\partial x^4} + [\pi E^s(R_o^3 + R_i^3)]\frac{\partial^4 w}{\partial x^4} + [P - 4\tau(R_o + R_i)]\frac{\partial^2 w}{\partial x^2} = 0$$

(6)

Solving equation (6) under given conditions at the fixed-free ends of the beam, the critical force of axial buckling is obtained as follows

$$P_{cr} = \frac{\eta\pi^2}{l^2}\left[\frac{\pi E(R_o^4-R_i^4)}{4} + \pi E^s(R_o^3 + R_i^3)\right] + 4\tau(R_o + R_i).$$

(7)

Here, l is the length of the nanotube and η is a dimensionless constant depending on the support conditions at the two end and for a fixed-free beam, which is our problem support condition, $\eta = 1/4$.

As an example, we consider the critical axial force of a fixed-free anodic alumina nanotube whit crystallographic of [111] direction. The material constants are given as $E = 70\,GPa$, $G = 27\,GPa$, $\rho = 2700\,kg/m^3$, $\tau = 0.9108\,N/m$ and $E^s = 5.1882\,N/m$ [5]. The critical axial forces are normalized with respect to the classical Euler's beam critical forces to evaluate the deviations and short comings of classical theories in accurately predicting the static behavior of nanoscaled structure elements. Figure 2 illustrates the size dependence in the buckling load ratio of Euler beam model including the surface effects in comparison to classical solutions of Euler beam model. Figure 2 shows the critical compressive force P_{cr}/P_{cr}^0 with respect to the outer radius of nanotube R_o performed for several cases of constantL/R_o ratios, where $P_{cr}^0 = [\pi^3 E(R_o^4 - R_i^4)]/4l^2$ is the critical axial force obtained by the classical Euler model. It is observed that as the outer radius increases its influence diminishes and the curves for various L/R_o tend to get closer to classical Euler beam critical axial load. In addition, the surface effects on the critical load are more prominent for a slender nanotube with a bigger ratio L/R_o.

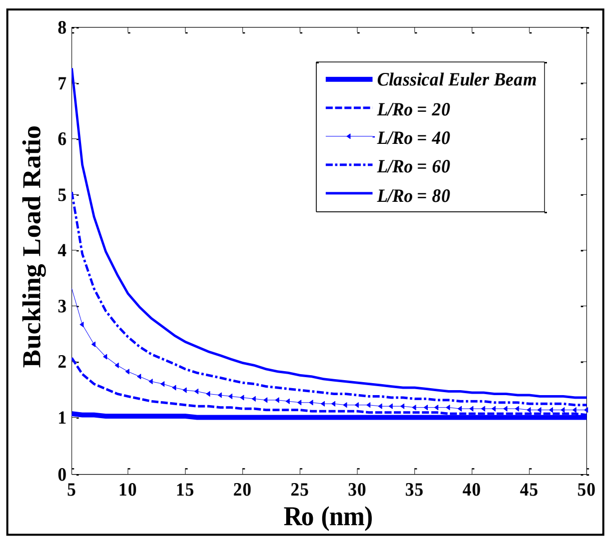

Fig. 2. (color online) Influence of surface effects and rotary inertia on the critical compressive load of the nanotube for $R_{in} = 3\,nm$.

Figure 3 displays the buckling load ratio P_{cr}/P_{cr}^0 of a nanotube for different ratio L/R_o. The influence of surface effects on the critical axial load becomes extremely significant as the ratio L/R_o increases in the range of nanometers.

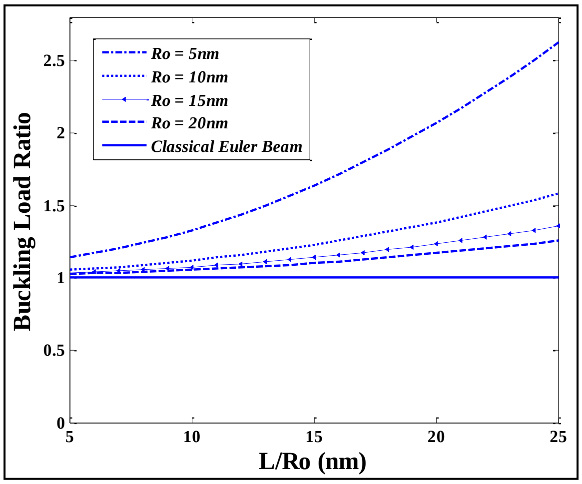

Fig. 3. (color online) The effect of aspect ratio L/R_i on the critical compressive load for $R_{in} = 3\ nm$.

CONCLUSIONS

In conclusion, this study predicts the size dependence in buckling analysis of nanotubes based on micro/nanoscaled Euler beam model. The outcome of the theoretical analysis represents that surface effects and rotary inertia can effect significantly on the critical buckling loads of nanotubes. The surface effects on the critical load tend to diminish when the volume of a nanotube increases while size ratios of its geometry remain constant. The surface effects with positive surface constants tend to increase the critical axial load in comparison to classical Euler beam model. The present study is crucial for design of nanoscaled chemical and biological measurement devices.

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